CC-14: Statistical Mechanics

Sem-6: January-June-2020

- **Slides made by: Somenath Jalal**
- **Department of Physics, Hooghly Women's College**
- **Reach me by: somenath.jalal@gmail.com**

Sem 6: CC-14: Statistical Mechanics

CC - XIV: STATISTICAL MECHANICS

(Credits: Theory-04, Practicals-02)

F.M. = 75 (Theory - 40, Practical – 20, Internal Assessment – 15)

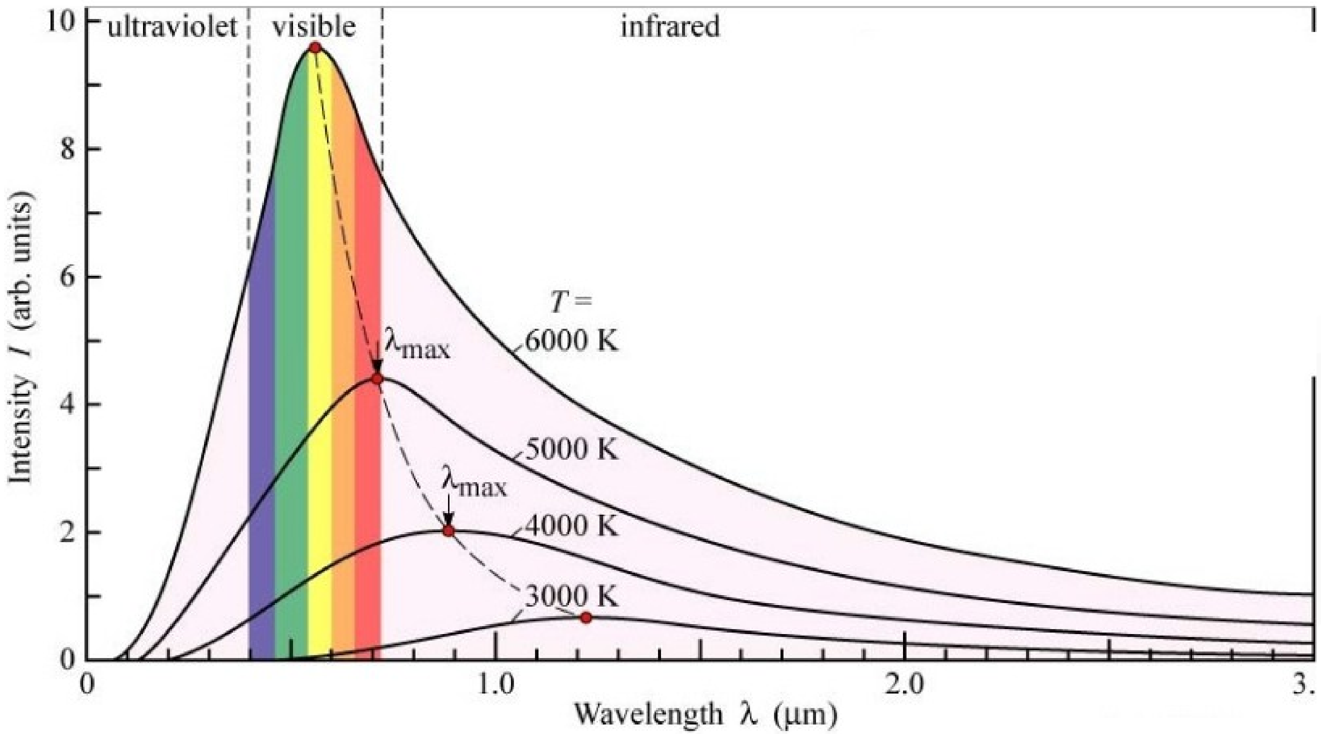
Internal Assessment [Class Attendance (Theory) – 05, Theory (Class Test/ Assignment/ Tutorial) – 05, Practical (Sessional Viva-voce) - 05]

Quantum Theory of Radiation: Spectral Distribution of Black Body Radiation. Planck's Quantum Postulates. Planck"s Law of Blackbody Radiation: Experimental Verification. Deduction of (1) Wien"s Distribution Law, (2) Rayleigh-Jeans Law, (3) Stefan-Boltzmann Law, (4) Wien"s Displacement law from Planck's law. (5 Lectures)

BLACK BODY RADIATION

You have seen how people were desperate to Fit the experimental curve with theory.

Theoretical Physicists wanted a single equation to fit the Experimental graph



Planck's quantum postulates

Let us consider the radiation is like collection of harmonic oscillators, and the energy of total N number of photons is given as

$$E = \sum_{s=1}^{N} n_s \epsilon_s \tag{14}$$

here n_s is number of photons in sth level and ϵ_s is energy of the sth level. Now one should remember here that the phontons are radiation particles can be considered as harmonic oscillators and can have energy 0 or $\hbar\omega$ or $2\hbar\omega$ etc. This is understood by the fact that photons are bosons and can stay in ϵ_s state in any number from zero (no photon) to infinity (very large number of photon). Thus n_s can chage values from 0 to ∞.

Thus the partition function is

$$Z = \sum_{n_1}\sum_{n_2}\cdots\sum_{n_N} e^{-\beta E}$$

Planck's quantum postulates

$$\langle n_7 \rangle = -\frac{1}{\beta}\frac{\partial}{\partial \epsilon_7} \log\left(\sum_{n_7=0}^{\infty} e^{-\beta n_7 \epsilon_7}\right)$$

In the mean time, let us calculate

$$\sum_{n_7=0}^{\infty} e^{-\beta n_7 \epsilon_7} = 1 + e^{-\beta\epsilon_7} + e^{-2\beta\epsilon_7} + e^{-3\beta\epsilon_7} \ldots = \frac{1}{1 - e^{-\beta\epsilon_7}}$$

In general for average number of particles in j^{th} level is

$$\langle n_j \rangle = \frac{1}{e^{\beta\epsilon_j} - 1}$$

This is Planck's distribution formula.

Planck's quantum postulates

Density of states ρ is independent of wave vector and proportional to volume V, and we get

$$\Delta n_x \Delta n_y \Delta n_z = \rho d^3k = \frac{L_x}{2\pi} dk_x \frac{L_y}{2\pi} dk_y \frac{L_z}{2\pi} dk_z = \frac{V}{8\pi^3} d^3k \tag{28}$$

Average number of photon per unit volume is

$$\langle n_j \rangle \ \rho \ d^3k = f(k) d^3k = f(k) 4\pi k^2 dk = \frac{1}{e^{\beta\hbar\omega} - 1} \frac{4\pi k^2 dk}{(2\pi)^3} \tag{29}$$

Now final form of photon energy density for both direction of polarization (factor 2) is

$$u(\omega, T) d\omega = \hbar\omega \ 2f(k) d^3k = \frac{\hbar}{\pi^2 c^2} \frac{\omega^3}{e^{\beta\hbar\omega} - 1} d\omega \tag{30}$$

Planck's quantum postulates

- **Can you write down the postulates?**
- **What is the new thing you learn in these exercise?**
- **What is average energy of photon? (½ k_BT ?)**

Plank's Formula for BBR

Look at the beauty:

S. N Bose was the first person to calculate the energy density from purely Mathematical point of view. He gave us Bose Statistics

$$u(\omega, T) = \frac{\hbar}{\pi^2 c^2} \frac{\omega^3}{e^{\beta \hbar \omega} - 1}$$

Popular stories: S N Bose invented Bose statistics formula when he was taking class to M.Sc students:

Deduction of Wien's Distribution Law

Let us first write the energy density in terms of ν from ω by using the relation $\omega = 2\pi\nu$. This we do to match the classical result taught in previous chapter/slides. Please have a look.

$$u(\omega, T) = \frac{\hbar}{\pi^2 c^2} \frac{\omega^3}{e^{\beta\hbar\omega} - 1} \quad (31)$$

$$u(\nu, T) = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{h\nu/k_B T} - 1} \quad (32)$$

remember to chage the $d\omega = 2\pi d\nu$

Wien's distribution law:

Wien's distributionlaw is valid for high frequency. $\nu >> 1$, we can write

$$\frac{1}{e^{h\nu/k_B T} - 1} = e^{-\frac{h\nu}{k_B T}}$$

thus the Wien's distribution law looks like:

$$u(\nu, T)d\nu = \frac{8\pi h}{c^3} \nu^3 e^{-\frac{h\nu}{k_B T}}$$

Rayleigh-Jeans law

Rayleigh-Jeans law

This law is valid for low frequency. $\nu << 1$.

$$\frac{1}{e^{h\nu/k_BT} - 1} = \frac{k_BT}{h\nu}$$

Thus from Eq. (32) we get

$$u(\nu, T)d\nu = \frac{8\pi h}{c^3}\nu^3\frac{k_BT}{h\nu} = \frac{8\pi h}{c^3}\nu^2 k_BT$$

Stefan-Boltzmann Law

Let us integrate the energy density function for all the frequency range

$$u(T) = \int_{\nu=0}^{\infty} u(\nu, T) d\nu = \int_{\nu=0}^{\infty} \frac{8\pi h}{c^3} \frac{\nu^3}{e^{h\nu/k_B T} - 1} d\nu \tag{37}$$

Take $\frac{h\nu}{k_B T} = x$, So we have $\nu = \frac{x k_B T}{h}$ and $d\nu = \frac{k_B T}{h} dx$. thus the integration becomes

$$u(T) = \frac{8\pi h}{c^3} \int_{\nu=0}^{\infty} \frac{\nu^3}{e^{h\nu/k_B T} - 1} d\nu = \frac{8\pi h}{c^3} (\frac{k_B T}{h})^3 \int_{x=0}^{\infty} \frac{x^3}{e^x - 1} (\frac{k_B T}{h}) dx \tag{38}$$

$$= \frac{8\pi h}{c^3} \left(\frac{k_B T}{h}\right)^4 \int_{x=0}^{\infty} \frac{x^3}{e^x - 1} dx \tag{39}$$

You are smart enough to do the integration. If you can not do it, dont not worry and whiten your hair. As we are already stressed due to Corona pandemic.

What we should understand as a physicist, the integration will give some constant value. But we got our energy density as a function of temperature..

$$u(T) = \sigma T^4 \tag{40}$$

Wien's displacement law

- **This is Home Work Problem :**

Thus we finish our Quantum theory of radiation chapter of Sem-6. Course: CC -14 : Statistical Mechanics.

Today is 15th April, 2020. Subho Naba-Barsha on Poila Boishakh.

Best of luck. Do not loose your life. Be happy. Do hard work.